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Journal of Geometry and Physics 58 (2008) 291–292

JOURNAL OF GEOMETRY AND PHYSICS

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Addendum

Addendum to "Homogeneous structures on three-dimensional Lorentzian manifolds" [J. Geom. Phys. 57 (2007) 1279–1291]

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Received 10 October 2007; accepted 23 October 2007
Available online 5 November 2007

We replace Lemma 2.3 in the paper by the following corrected version:

Lemma 2.3. *Let (M, g) be a connected, simply connected and complete pseudo-Riemannian manifold. If (M, g) admits a global pseudo-orthonormal frame field $\{e_1, \ldots, e_n\}$ and some constants B_{ijk}, satisfying $\nabla_{e_i} e_j = \sum_k \varepsilon_j B_{ijk} e_k$ for all i, j, then M has a Lie group structure, unique up to isomorphisms, such that e_i are left-invariant vector fields and g is left-invariant.*

Proof. We define a tensor field T of type (1, 2) on M, by

$$T e_i := \frac{1}{2} \sum_{jk} B_{ijk} e_j \wedge e_k,$$

for all i, where $e_j \wedge e_k(X) = g(e_j, X)e_k - g(e_k, X)e_j$. From the constancy of B_{ijk} it easily follows that T is a homogeneous pseudo-Riemannian structure on (M, g). Moreover, $T_{e_i} e_j = \nabla_{e_i} e_j$ and so, $\tilde{\nabla}_{e_i} e_j = 0$, for all i, j, where $\tilde{\nabla}$ is the canonical connection associated to T.

Being the canonical connection of an affine homogeneous space, $\tilde{\nabla}$ is complete (Chapter X, Corollary 2.5 of [1]). Then, all vector fields e_i are complete, since $\tilde{\nabla}_{e_i} e_i = 0$ and hence, their integral curves are geodesics with respect to $\tilde{\nabla}$. Moreover,

$$[e_i, e_j] = \nabla_{e_i} e_j - \nabla_{e_j} e_i = \sum_k c_{ijk} e_k,$$

where $c_{ijk} = \varepsilon_j B_{ijk} - \varepsilon_i B_{jik}$ is a constant for all i, j, k. Therefore, Proposition 1.9 of [2] implies that for a fixed point $p \in M$, the manifold M has a unique Lie group structure, such that p is the identity and all vector fields e_i are left-invariant. The fact that $\{e_i\}$ is a pseudo-orthonormal basis of left-invariant vector fields then yields at once that g is left-invariant. □

DOI of original article: 10.1016/j.geomphys.2006.10.005.

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0393-0440/$ - see front matter
doi:10.1016/j.geomphys.2007.10.006

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